Perspective as Symbolic Form

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"Item Perspectiva ist ein lateinisch Wort, bedeut ein Durchsehung" ("Perspectiva is a Latin word which means ‘seeing through.’"). This is how Dürer sought to explain the concept of perspective. And although this *lateinisch Wort* was used already by Boethius, and did not originally bear so precise a meaning, we shall nevertheless adopt in essence Dürer’s definition. We shall speak of a fully “perspectival” view of space not when mere isolated objects, such as houses or furniture, arc represented in “foreshortening,” but rather only when the entire picture has been transformed — to cite another Renaissance theoretician — into a “window,” and when we are meant to believe we are looking through this window into a space. The material surface upon which the individual figures or objects are drawn or painted or carved is thus negated, and instead reinterpreted as a mere “picture plane.” Upon this picture plane is projected the spatial continuum which is seen through it and which is understood to contain all the various individual objects.

So far it does not matter whether this projection is determined by an immediate sensory impression or by a more or less “correct” geometrical construction. This correct construction was in fact invented in the Renaissance, and although later subjected to
various technical improvements and simplifications, it nevertheless remained in its premises and goals unchanged to the time of Desargues. It is most simply explained as follows: I imagine the picture — in accord with the “window” definition — as a planar cross section through the so-called visual pyramid; the apex of this pyramid is the eye, which is then connected with individual points within the space to be represented. Because the relative position of these “visual rays” determines the apparent position of the corresponding points in the visual image, I need only draw the entire system in plan and elevation in order to determine the figure appearing on the intersecting surface. The plan yields the width, the elevation yields the height; and if I combine these values on a third drawing, I will obtain the desired perspectival projection (Figure 1).

In a picture constructed this way — that is, by means of what Dürer called a “planar, transparent intersection of all those rays that fall from the eye onto the object it sees” — the following laws are valid. First, all perpendiculars or “orthogonals” meet at the so-called central vanishing point, which is determined by the perpendicular drawn from the eye to the picture plane. Second, all parallels, in whatever direction they lie, have a common vanishing point. If they lie in a horizontal plane, then their vanishing point lies always on the so-called horizon, that is, on the horizontal line through the central vanishing point. If, moreover, they happen to form a 45-degree angle with the picture plane, then the distance between their vanishing point and the central vanishing point is equal to the distance between the eye and the picture plane. Finally, equal dimensions diminish progressively as they recede in space, so that any portion of the picture — assuming that the location of the eye is known — is calculable from the preceding or following portion (see Figure 7).

In order to guarantee a fully rational — that is, infinite, un-
changing and homogeneous – space, this "central perspective" makes two tacit but essential assumptions: first, that we see with a single and immobile eye, and second, that the planar cross section of the visual pyramid can pass for an adequate reproduction of our optical image. In fact these two premises are rather bold abstractions from reality, if by "reality" we mean the actual subjective optical impression. For the structure of an infinite, unchanging and homogeneous space – in short, a purely mathematical
space – is quite unlike the structure of psychophysiological space: "Perception does not know the concept of infinity; from the very outset it is confined within certain spatial limits imposed by our faculty of perception. And in connection with perceptual space we can no more speak of homogeneity than of infinity. The ultimate basis of the homogeneity of geometric space is that all its elements, the 'points' which are joined in it, are mere determinations of position, possessing no independent content of their own outside of this relation, this position which they occupy in relation to each other. Their reality is exhausted in their reciprocal relation: it is a purely functional and not a substantial reality. Because fundamentally these points are devoid of all content, because they have become mere expressions of ideal relations, they can raise no question of a diversity in content. Their homogeneity signifies nothing other than this similarity of structure, grounded in their common logical function, their common ideal purpose and meaning. Hence homogeneous space is never given space, but space produced by construction; and indeed the geometrical concept of homogeneity can be expressed by the postulate that from every point in space it must be possible to draw similar figures in all directions and magnitudes. Nowhere in the space of immediate perception can this postulate be fulfilled. Here there is no strict homogeneity of position and direction; each place has its own mode and its own value. Visual space and tactical space [Tastrum] are both anisotropic and unhomogeneous in contrast to the metric space of Euclidean geometry: 'the main directions of organization – before–behind, above–below, right–left – are dissimilar in both physiological spaces.' "

Exact perspectival construction is a systematic abstraction from the structure of this psychophysiological space. For it is not only the effect of perspectival construction, but indeed its intended purpose, to realize in the representation of space pre-
cisely that homogeneity and boundlessness foreign to the direct experience of that space. In a sense, perspective transforms psychophysiological space into mathematical space. It negates the differences between front and back, between right and left, between bodies and intervening space ("empty" space), so that the sum of all the parts of space and all its contents are absorbed into a single "quantum continuum." It forgets that we see not with a single fixed eye but with two constantly moving eyes, resulting in a spheroidal field of vision. It takes no account of the enormous difference between the psychologically conditioned "visual image" through which the visible world is brought to our consciousness, and the mechanically conditioned "retinal image" which paints itself upon our physical eye. For a peculiar stabilizing tendency within our consciousness – promoted by the cooperation of vision with the tactile sense – ascribes to perceived objects a definite and proper size and form, and thus tends not to take notice, at least not full notice, of the distortions which these sizes and forms suffer on the retina. Finally, perspectival construction ignores the crucial circumstance that this retinal image – entirely apart from its subsequent psychological "interpretation," and even apart from the fact that the eyes move – is a projection not on a flat but on a concave surface. Thus already on this lowest, still prepsychological level of facts there is a fundamental discrepancy between "reality" and its construction. This is also true, of course, for the entirely analogous operation of the camera.

If, to choose a very simple example, a line is divided so that its three sections \(a\), \(b\) and \(c\) subtend equal angles, these objectively unequal sections will be represented on a concave surface (like the retina) as approximately equal lengths; whereas if projected on a flat surface they will appear, as before, as unequal lengths (Figure 2). This is the source of those marginal distortions
which are most familiar to us from photography, but which also distinguish the perspectively constructed image from the retinal image. These distortions can be mathematically expressed as the discrepancy between, on the one hand, the ratio of the visual angles and, on the other hand, the ratio of the linear sections produced by projection upon a flat surface. The wider the total or composite visual angle—that is, the smaller the ratio between the distance from eye to image and the size of the image—the more pronounced the distortion. But alongside this purely quantitative discrepancy between retinal image and perspectival representation, which was recognized already in the early Renaissance, there is as well a formal discrepancy. This latter follows, in the first place, from the movement of the gaze, and in the second place, once again, from the curvature of the retina: for while perspective projects straight lines as straight lines, our eye perceives them (from the center of projection) as convex curves. A normal checkerboard pattern appears at close range to swell out in the
form of a shield; an objectively curved checkerboard, by the same token, will straighten itself out. The orthogonals of a building, which in normal perspectival construction appear straight, would, if they were to correspond to the factual retinal image, have to be drawn as curves. Strictly speaking, even the verticals would have to submit to some bending (pace Guido Hauck, whose drawing is reproduced as Figure 3).

This curvature of the optical image has been observed twice in modern times: by the great psychologists and physicists at the end of the last century,9 but also (and this has apparently not been remarked upon until now) by the great astronomers and mathematicians at the beginning of the seventeenth century. We should recall above all the words of the remarkable Wilhelm Schickhardt, a cousin of the Württemberg architect and Italian traveler, Heinrich Schickhardt: “I say that all lines, even the straightest, which do not stand directe contra pupillam [directly in front of the eye]... necessarily appear somewhat bent. Never-

![Figure 3](image-url)

**Figure 3.** Hall of pillars constructed according to “subjective” or curved perspective (left) and according to schematic or linear perspective (right).
(After Guido Hauck.)
theless, no painter believes this; this is why they paint the straight sides of a building with straight lines, even though according to the true art of perspective this is incorrect.... Crack that nut, you artists!” This was endorsed by none other than Kepler, at least insofar as he admitted the possibility that the objectively straight tail of a comet or the objectively straight trajectory of a meteor is subjectively perceived as a curve. What is most interesting is that Kepler fully recognized that he had originally overlooked or even denied these illusory curves only because he had been schooled in linear perspective. He had been led by the rules of painterly perspective to believe that straight is always seen as straight, without stopping to consider that the eye in fact projects not onto a \textit{plana tabella} but onto the inner surface of a sphere. And indeed, if even today only a very few of us have perceived these curvatures, that too is surely in part due to our habituation—further reinforced by looking at photographs—to linear perspectival construction: a construction that is itself comprehensible only for a quite specific, indeed specifically modern, sense of space, or if you will, sense of the world.

Thus in an epoch whose perception was governed by a conception of space expressed by strict linear perspective, the curvatures of our, so to speak, spheroidal optical world had to be rediscovered. However, in a time that was accustomed to seeing perspectivally—but not in linear perspective—these curvatures were simply taken for granted: that is, in antiquity. In antique optics and art theory (as well as in philosophy, although here only in the form of analogies) we constantly encounter the observations that straight lines are seen as curved and curved lines as straight; that columns must be subjected to \textit{entasis} (usually relatively weak, of course, in classical times) in order not to appear bent; that epistyle and stylobate must be built curved in order to avoid the impression of sagging. And, indeed, the familiar
curvatures of the Doric temple attest to the practical consequences of such findings. Antique optics, which brought all these insights to fruition, was thus in its first principles quite antithetical to linear perspective. And if it did understand so clearly the spherical distortions of form, this only follows from (or at least corresponds to) its still more momentous recognition of the distortions of magnitude. For here, too, antique optics fit its theory more snugly to the factual structure of the subjective optical impression than did Renaissance perspective. Because it conceived of the field of vision as a sphere, antique optics maintained, always and without exception, that apparent magnitudes (that is, projections of objects onto that spherical field of vision) are determined not by the distances of the objects from the eye, but rather exclusively by the width of the angles of vision. Thus the relationship between the magnitudes of objects is, strictly speaking, expressible only in degrees of angle or arc, and not in simple measures of length. Indeed Euclid’s Eighth Theorem explicitly preempts any opposing view. Euclid states that the apparent difference between two equal magnitudes perceived from unequal distances is determined not by the ratio of these distances, but rather by the far less discrepant ratio of the angles of vision (Figure 4). This is diametrically opposed to the doctrine behind modern perspectival construction, familiar in the formula of Jean Pélerin, known as Viator: “Les quantités et les distances ont concordables différences” (“The quantities and the distances vary proportionally”). And perhaps it is more than mere accident that in Renaissance paraphrases of Euclid, indeed even in translations, precisely this Eighth Theorem was either entirely suppressed or “emended” until it lost its original meaning. Evidently, the contradiction was felt between Euclid’s perspectiva naturalis or communis, which sought simply to formulate mathematically the laws of natural vision (and so linked apparent size to the visual angle),
Figure 4. Contrast between the "linear perspectival" and "angle-perspectival" constructions: in linear perspective (left), the apparent sizes (HS and JS) are inversely proportional to the distances (AB and AD); in angle perspective (right), the apparent sizes (β and a + β) are not inversely proportional to the distances (2b and b).

and the *perspectiva artificialis* developed in the meantime, which on the contrary tried to provide a serviceable method for constructing images on two-dimensional surfaces. Clearly, this contradiction could be resolved only by abandoning the angle axiom; to recognize the axiom is to expose the creation of a perspectival image as, strictly speaking, an impossible task, for a sphere obviously cannot be unrolled on a surface.
NOTES


20. This is one of the implications of Klein’s thinking on perspective; see “Pomponius Gauricus on Perspective.”
21. See the radical nominalist position of Nelson Goodman in Languages of Art (Indianapolis: Hackett, 1976), pp. 10-19, especially p. 16 and n. 17, with references to various like-minded thinkers.
24. Damisch, L’Origine de la perspective, p. 32ff.

PERSPECTIVE AS SYMBOLIC FORM

2. Boethius, Analys. poster. Aristot. Interpretatio 1.7 and 1.10, in Opera (Basel, 1570), pp. 527 and 538; perspectiva is characterized in both passages as a subdiscipline of geometry.
3. The word ought to be derived not from perspicere meaning “to see through,” but from perspicere meaning “to see clearly”; thus it amounts to a literal translation of the Greek term optike. Dürrer’s interpretation is based already on the modern definition and construction of the image as a cross section...
through the visual pyramid. Felix Witting, on the other hand, detected in the transformation of the Italian perspettiva into prospettiva a kind of protest against this understanding of the image ("the former is reminiscent of Brunelleschi's 'punto dove percorre la occhio,' whereas the latter suggests only a seeing forward," Von Kunst und Christentum [Strassburg, 1903], p. 106). This is more than doubtful, for it is precisely the most rigorous theoreticians of the cross-section method, such as Piero della Francesca, who use the term prospettiva. At most we can grant that prospettiva implies more strongly the idea of the artistic achievement (namely the conquest of spatial depth), while perspettiva evokes rather the mathematical procedure. A purely phonetic consideration must have then favored the triumph of the term prospettiva, to wit, an aversion to the sequence of consonants "rsp."

4. Leon Battista Alberti, Della pittura, in Kleine kunsthistorische Schriften, Quellenschriften für Kunstgeschichte und Kunsttechnik des Mittelalters und der Renaissance, no. 11, ed. Hubert Janitschek (Vienna, 1877), p. 79: "scrisso una quadroangula ... e il quale reputo essere una fenestra aperta per donde io vidi quello che qui visi dipinto" (On Painting, trans. John R. Spencer [New Haven: Yale University Press, 1966], p. 56: "I inscribe a quadrangle... which is considered to be an open window through which I see what I want to paint"). See also Leonardo (Jean Paul Richter, The Literary Works of Leonardo da Vinci [London, 1833], no. 83), where the same analogy to a "pariete di vetro," or pane of glass, is drawn.

5. Already Lessing, in the ninth of his Antiquarische Briefe, distinguished between a broader and a narrower meaning of perspective. In the broader sense perspective is "the science of representing objects on a surface just as they would appear to our eye at a certain distance... Not to credit the ancients with perspective in this sense would be rather foolish. For it would mean depriving them not only of perspective but of the entire art of drawing, an art which they had quite mastered. No one could maintain this. Rather, when one contests the antique claim to perspective, it is in this narrower sense, the sense in which artists take the word. For artists, perspective is the science of representing a number of objects together with the space around them, just as these objects, dispersed among various planes of the space, together with their space, would
Essentially, then, we are adopting Lessing's second definition, only that we formulate it a little more liberally by dropping the condition of the rigorously maintained single point of view. For unlike Lessing we accept late Hellenistic and Greco-Roman paintings as already authentically "perspectival." For us perspective is, quite precisely, the capacity to represent a number of objects together with a part of the space around them in such a way that the conception of the material picture support is completely supplanted by the conception of a transparent plane through which we believe we are looking into an imaginary space. This space comprises the entirety of the objects in apparent recession into depth, and is not bounded by the edges of the picture, but rather only cut off.

There are of course a multitude of transitional cases between mere "foreshortening" (which for its part does represent the necessary first step and precondition for the development of a true perspectival conception of space) and something recognizable as perspective in this sense. An example of such a transitional case are those well-known southern Italian vases which show a figure or even several figures assembled in a foreshortened aedicule. This approximates true perspective insofar as a greater spatial construct already contains within it a number of individual bodies; but this greater spatial construct is itself still offered up as an isolated object, upon a picture support which retains its materiality. Instead, the entire surface of the painting would have to be transformed into a projective plane for a perspectival illusion of the entire space.

6. Lange and Fuhse, Dürers schriftlicher Nachlass, p. 195, l. 15: "Ein ebene durchsichtige Abschneidung aller der Streimlinien, die aus dem Auge fallen auf die Ding, die es sicht."

7. Ernst Cassirer, Philosophie der symbolischen Formen, vol. 2: Das mythische Denken (Berlin: B. Cassirer, 1925), p. 107f. (Philosophy of Symbolic Forms, vol. 2: Mythical Thought, trans. Ralph Manheim [New Haven: Yale University Press, 1955], pp. 83–84. [In the last sentence of the passage Cassirer quotes Ernst Mach. –TK]). For the psychophysiological view of space, of course, the distinc-
tion between solid bodies and the medium of open space surrounding them is sharper than that between “front” and “back,” etc. For immediate and mathematically unrationalized perception, empty space is qualitatively altogether different from “objects.” On this subject, see E. R. Jaensch, Über die Wahrnehmung des Raumes, Zeitschrift für Psychologie, supplement 6 (Leipzig: Barth, 1911), sec. 1, ch. 6: “Zur Phänomenologie des leeren Raumes.”

8. On the phenomenon of marginal distortions, see above all Guido Hauck, Die subjektive Perspektive und die horizontalen Curvenuren des Dorischen Styls (Stuttgart, 1879), esp. p. 51f., and “Die malerische Perspektive.” Wochenblatt für Architekten und Ingenieuren 4 (1882). On the historical aspects, see Hans Schuritz, Die Perspektive in der Kunst Dürers (Frankfurt: H. Keller, 1919), p. 11ff., among others. This problem was rather disconcerting for Renaissance theoreticians because marginal distortions expose an undeniable contradiction between the construction and the actual visual impression; indeed, under some circumstances the “foreshortened” dimensions can exceed the “unforeshortened.” The differences in opinion are nevertheless instructive. The rigorous Piero della Francesca, for one, decides the dispute between perspective and reality without hesitation in favor of the former (De prospectiva pingendi, ed. C. Winterberg [Strassburg, 1899], p. xxxi). Piero recognizes the fact of marginal distortions and adduces the example (used by Hauck as well as by Leonardo: see Richter, Leonardo da Vinci, no. 544) of the exact perspectival construction of a frontal portico, or any comparable structure with a row of objectively equal elements, in which the breadth of the elements increases toward the edges (Figure 9). But so far from proposing a remedy, Piero proves rather that it must be so. One may marvel at this, he says; and yet “io intendo di dimostrare così essere e doversi fare.” Then follows the strictly geometrical proof (which is, of course, very easy, for precisely the premise upon which the proof rests, namely the planar section of the visual pyramid, necessarily entails marginal distortions) and, introduced here not unintentionally, a long encomium of perspective. The conciliatory Ignazio Danti (in Jacopo Barozzi da Vignola, Le Due regole della prospectiva pratica, edited with commentary by Danti [Rome, 1583]) denies marginal distortions altogether when they are less blatant (see, for example, p. 62); he then recom-
Figure 9. Marginal distortions in a linear perspectival construction of a row of equally thick columns: $a = \delta < \beta$, but $AB = EF > CD$. (After Leonardo.)
Leonardo emphasizes, again anticipating the results of modern psychological research, the especially strong power of illusion of pictures with short perpendicular distances, which rests on the rapidity of the foreshortening and the concomitant expansion of the depth intervals (of course, with the restriction that the illusion is only effective if the eye of the beholder remains fixed exactly at the center of projection, for only then can the disproporzioni disappear). Thus the artist ought in general to avoid short perpendicular distances: "If you want to represent an object near to you which is to have the effect of nature, it is impossible that your perspective should not look wrong, with every false relation and disagreement of proportion that can be imagined in a wretched work, unless the spectator, when he looks at it, has his eye at the very distance and height and direction where the eye or the point of sight was placed in doing this perspective." (One must thus fix the eye of the beholder by means of a small peephole.) "If you do this, beyond all doubt your work, if it is correct as to light and shade, will have the effect of nature; nay you will hardly persuade yourself that those objects are painted; otherwise do not trouble yourself about it, unless indeed you make your view at least twenty times as far off as the greatest width or height of the objects represented, and this will satisfy any spectator placed anywhere opposite to the picture." And in Richter, nos. 107-109, occurs the exceedingly perspicacious justification for that apparent canceling out of the marginal distortions when the eye is fixed at the center of projection (see, by contrast, Jaensch's quite unsatisfactory explanation of the phenomenon, in Über die Wahrnehmung des Raumes, p. 160): it consists in a collaboration between perspettiva naturale — that is, the alteration that the dimensions of the panel or wall undergo when observed by the beholder — and perspettiva accidentale — that is, the alteration that the dimensions of the natural object already suffered when the painter observed and reproduced it. These two perspectives work in exactly contrary senses, for perspettiva accidentale, as a consequence of planar perspectival construction, broadens the objects off to the sides, whereas perspettiva naturale, as a consequence of the diminution of the angle of vision toward the edges, narrows the margins of the panel or wall (see Figure 9). Thus the two perspectives cancel each other out when the eye is situated exactly in the center of projec-
tion, for then the edges of the panel recede with respect to the central parts, by virtue of natural perspective, in exactly the same proportion that they expand by virtue of accidental perspective. Even in this discussion, however, Leonardo again and again recommends avoiding just such a perspettiva composta (the term is especially clearly developed in Richter, no. 90) resting on the mutual cancelation of the two perspectives, and instead making do with a perspettiva semplice, in which the perpendicular distance is set so large that the marginal distortions have no importance; such a perspective remains palatable regardless of where the beholder stands.

Jaensch seems to have overlooked all these observations of the Italian theoreticians, particularly those of Leonardo, for he claims (p. 159ff.) that Dürer and the masters of the early Renaissance had "not noticed" marginal distortions (which Jaensch, moreover, by neglecting the curvature of the retina, derives exclusively from the discrepancy between the apparent sizes and the size of the retinal image; this is why he treats both the perspectively constructed image and the photograph as equivalent to the retinal image). According to Jaensch, because they ignored the distortions, they systematically demanded from their representations that powerful illusionistic effect generated precisely by the apparent deformations of the pictures with short perpendicular distances. Leonardo is for Jaensch a prime witness of this desire (in and of itself undeniable) for strong plastic illusion ("rilievo"). And yet it was precisely Leonardo who most thoroughly investigated the phenomenon of marginal distortions, and who most decisively warned against constructions with short distances. The Italians, furthermore, for whom this rilievo was undoubtedly at least as desirable a goal as for the northerners, in general and on principle preferred greater distances to shorter distances, not only in theory but also in practice. It is no accident that Jaensch draws his concrete examples entirely from northern art (Dürer, Roger van der Weyden, Dirk Bouts). As a matter of fact, construction with a short perpendicular distance was employed not to realize general Renaissance ideals of strong plasticity, but rather to realize the peculiarly Northern ideal of an impression of a quite specifically interior space, that is, an impression of including the beholder within the represented space; see further, p. 69 and note 69, below.
9. See specifically Hermann von Helmholtz, *Handbuch der physiologischen Optik* (Hamburg & Leipzig: Voss, 1910), vol. 3, p. 151 (Physiological Optics [New York: Dover, 1960], vol. 3, pp. 178-87); Hauck, *Die subjektive Perspektive*; Peter, “Studien über die Struktur des Sehraums.” Especially instructive is the counter-proof, the so-called curved-path experiment. If a number of mobile individual points (small lights or the like) are ordered in two rows leading into depth in such a way that a subjective impression of parallel straight lines ensues, then the objectively resulting form will be concave, trumpet-like (see Franz Hillebrand, “Theorie der scheinbaren Grösse bei binocularem Sehen,” *Denkschriften der Kaiserlichen Akademie der Wissenschaften, Mathematisch-Naturwissenschaftliche Klasse*, no. 72 [1902], pp. 255-307; the critiques of his arguments — see among others Walther Poppelreuter, “Beiträge zur Raumpychologie,” *Zeitschrift für Psychologie* 58 [1911], pp. 200-62 — do not impinge upon matters essential to us here).

10. Wilhelm Schickhardt, professor of oriental languages and of mathematics at Tübingen, as well as a dilettante woodcut artist and engraver, wrote a small work about a meteor observed on November 7, 1623, in various places in southern Germany. This work, composed very hastily to preserve its topicality, provoked a number of attacks. To refute these attacks, Schickhardt prepared in the following year a most interesting and in part quite spirited and humorous pamphlet; interesting, for example, for its position on the question whether and how far the prophetic meaning of such celestial phenomena could be clarified. The pamphlet was entitled “The Ball of Light, treating, as a primer on the miraculous light which recently appeared, not only that one in specie but also similar meteors in generis... that is, a kind of German Optics.” In this book is found the following remarks as a proof that the trajectory (“Durchschuss”) of the celestial body in question, even though it may appear subjectively to be curved, is in fact objectively nearly straight (p. 96ff.; Figure 10): “In any case, even if it was somewhat curved, it cannot have been very noticeable, but rather must have happened only apparenter et optice; and vision must have been deceived in the following two ways. First, I say that all lines, even the straightest, which do not stand directe contra pupillam (directly in front of the eye), or go through
its axis, necessarily appear somewhat bent. Nevertheless, no painter believes this; this is why they paint the straight sides of a building with straight lines, even though according to the true art of perspective this is incorrect. Furthermore — and this will appear absurd even to the scholars of optics themselves, who believe that omnes perpendiculares appare rectas [all perpendiculares appear straight], which, strictly speaking, is not true — it is evident and undeniable that parallel lines appear to the eye to converge and ultimately meet at a single point. It can be observed, for instance, that although long rooms or cloisters may be of objectively equal breadth, they nevertheless appear progressively smaller and narrower. Let us now take as an example a square or a quadrilateral $BDKM$ with the eye in the middle, at $G$; the four edges, because they are all in front of the eye, must diminish as they approach the four external points $A$, $E$, $I$ and $N$. Or to put it more intelligibly: the nearer an object, the larger it appears, and conversely the further away it is, the smaller it appears; this can be shown with any finger, which close to the eye covers an entire village, further off hardly a single field. For, as in the figure above, the median lines $CL$ and $FH$ are the nearest to the eye (since they pass through it), they must appear larger; whereas the sides $BD$, $DM$, $MK$, $KB$ are further from it, and so must appear smaller. Thus the sides

![Figure 10. Proof of the "optical curves." (After Schickhardt.)](image-url)
become narrower and necessarily curved; not like a roof, to be sure, so as to produce a sharp angle at the points C, F, H, L, but rather gently and gradually, indeed unnoticeably, something like a belly, as is appropriate for such an arc. Thus it is certainly not true to nature when the painter draws a straight wall on paper with straight lines. Crack that nut, you artists!

Similar problems were addressed by, besides Kepler (see the following note), Franciscus Aguilonius, Opticorum libri sex (Antwerp, 1613), 4.44, p. 265, except that he addresses not so much the bending as the refraction of the lines: “huius difficultati occurrendum erit plane asserendo omnium linearum, quae horizonti aequilibres sunt, solam illum, quae pari est cum horizonte altitudine, rectam videri, ceteras vero inflexas, ac illas quidem, quae supra horizontem eminent, ab illo puncto, in quod aspectus maxime dirigitur, utrinque procedere, quae autem infra horizontem procumbunt, utrinque secundum aspectum attolligatur...rursus e perpendicularibus medium illum, in quam obtutus directe intenditur, videri rectam, ceteras autem superne atque inferne inclinari eaque ratione inflexas videri” (“This difficulty will be encountered in determining clearly which of all lines are horizontal: that one alone, which is equal in height to the horizontal, appears straight, but the others as inflexas [which in the author’s usage means “broken,” whereas “bent” is rendered as “incurvus”], and those lines moreover which rise out above the horizontal, from that point on which one’s gaze is especially fixed, from either side fall forward; and again, the line at that middle point of the perpendiculars, on which one’s eye is directly fixed, seems straight, whereas the others bend out above and below and in that way appear broken”).

11. Johannes Kepler, Appendix hyperaspistis 19, in Opera omnia, 8 vols., ed. Christian Frisch (Frankfurt & Erlangen, 1858-1871), vol. 7, p. 279; on p. 292 of the same work he reproduces the passage from Schickhardt in the previous note, although not in its entirety: “Fateor, non omnino verum est, quod negavi, ea quae sunt recta, non posse cito refractionem in coelo representari curva, vel cum parallaxi, vel etiam sine eo. Cum hanc negationem perscriberem, versabatur in animo projectiones visibilium rerum in planum, et notae sunt praecipue graphicæ seu perspectivos, quæ quantumcumque diversitate propinquitatís terminorum alius ejus rectae semper ejus rectae vestigia representatoria super plano picturæ in rectam itidem
I confess that it is not entirely true, as I have denied, that those lines which are straight cannot apart from refraction be represented in the sky as curved, or similarly with parallel lines or other cases. Since I have retracted this denial, it used to be that projections of visible things were treated in the mind as though projected on a plane surface, and perceptions were noted as graphic and perspectival which, according to the distance from terminal objects, order the represented traces as straight over the surface of the picture in a straight line. But our vision does not in fact have a plane surface like a tablet, on which it contemplates the painting of a half-sphere, but rather that image of the sky, against which it sees comets, it produces in itself as spherical by natural instinct of vision; and if the image of objects is projected into a concave sphere with straight lines of extension, the representations of those lines will be not straight, but in fact curved, just as in the circle, no doubt, of the greatest sphere, if it is seen from its center, as we teach about projection in circular astrolabes”). (See also Kepler's Paralipomena in Vitellionem 3.2.7 [Opera, vol. 2, p. 167]; the spherical form of the eye corresponds to the spherical form of the visual image, and the estimation of size is carried out by comparing the entire surface of the sphere to the respective portion of it: “mundus vero hic asceptibilis et ipse concavus et rotundus est, et quidquid de hemisphaerio aut eo amplius intuemur uno obtuto, id pars est huius roountatis. Consentamentum igitur est, proportionem singularum rerum ad totum hemisphaeirum ostiumri e visu proportione speciei ingressae ad hemisphaerium oculi. Atque hic est vulgo dictus angulus visiorius” (“This world is indeed visible and is itself concave and round, and whatever in the hemisphere we perceive as greater than it in a single glance, this is equal in its rotundity. It therefore follows that the proportion of individual things to the entire hemisphere is estimated by vision in proportion to the image entering in upon the hemisphere of the eye.

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lineam ordinant. At vero visus noster nullum planum pro tabella habet, in qua contemplatur picturam hemisphaerii, sed faciem illam coeli, super qua videt cometas, imaginatur sibi sphaericam instictu naturali visionis, in concavum vero sphaericum si projiciatur puctura rerum rectis lineis extensum, earum vestigia non erunt lineae rectae, sed multeque curvae, circuli nimium maximi sphaerar, si visus in ejus centro sit, ut docentur de projectione circularum in astrolabium”
And this is commonly called the angle of vision”). This theory of the estimation of size is entirely in accord with Alhazen, Optica 2.37 and Vitellio, Perspectiva communis 4.17; on the premise of the spherical field of vision, see note 13, below.) Thus because the bending of the optical image is for Kepler grounded only in, as it were, an erroneous localization of the visual impression, but not in its actual structure, he must necessarily reject Schickhardt’s view that even painters ought to represent all straight lines as bent: “Confundit Schickardus separanda: cœunt versus punctum visionis in plano picturae omnia rectarum realium, quae radio visionis paralleleae exunt, vertigio in plano picturae, vicissim curvantur non super plano picturas, sed in imaginazione visi hemisphaerii omnes rectae reales et inter se parallelae, et curvantur versus utrumque latus rectae ex oculo in sese perpendicularis, curvantur inquam neque realiter neque pictoria, sed apparenter solum, id est videntur curvari. Quid igitur quaeres, numquid ea pictura, quae exaratur in plano, repraesentatio est apparentiae hujus parallelarum? Est, inquam, et non est. Nam quatenus consideramus lineas versus utrumque latus curvari, oculi radium cogitatione perpendiculariter facimus incidere in medium parallelarum, oculum ipsum seorsum collocamus extra parallelas. Cum autem omnis pictura in plano sit angusta pars hemisphaerii aspectabilis, certe planum objectum perpendiculariter radii visoria jam dicta nullam complectetur partem apparentiae curvatarum utrinque parallelarum: quippe cum apparentia hoc se recipiant ad utrumque latus finemque hemisphaerii visivi. Quando vero radium visirum cogitatione dirigimus in alterum punctorum, ac in quos appetenter cœunt parallelae, sic ut is radius visivus sit quasi medius parallelarum: tunc pictura in plano artificiosa est huius visionis genuina et propria representatio. At neutobique consenteantem est naturae, ut pingantur curvae, quod fol. 98 desiderabat scriptor” (“Schickhardt confuses things that ought to be kept separate. All representations of straight lines in the plane of the picture that go out parallel to the angle of vision converge on a point of vision in the plane of the picture. Conversely, all straight lines parallel to themselves are curved not over the plane of the picture, but in the imagination of the visible hemisphere, and they are curved toward either side straight from the eye perpendicular to themselves, and thus they are curved neither in reality nor pictorially, but only seemingly; that is, they only appear to be curved. Why, therefore, do you ask
why it is that, in those pictures which are executed in a plane, there is a
representation of the appearance of these lines as parallel? There is, and there
is not. For to the extent that we consider the lines to be curved toward each
other, we cause the angle of the eye to fall in thought perpendicularly in the
middle of the parallel lines, and we locate the eye itself apart outside the parallel
lines. Moreover, since the entire picture in the plane is in the narrow part of
the visible hemisphere, surely the plane projecting perpendicularly from the
aforesaid angle of vision will embrace no part of the appearance of curved or
parallel lines, especially since this appearance is received on either side and the
end of the hemisphere of vision. Indeed when we direct the angle of vision in
thought to any other point where the parallel lines apparently converge, in such
a way that this angle of vision is in the middle as it were of the parallel lines,
then the artificial picture in the plane is a genuine and proper representation
of this vision. But it is never consistent with nature to depict them as curved,
as the author [Schickhardt] was desiring).

12. That right angles appear round when seen from a distance (and that,
by the same token, an arc becomes under certain conditions a straight line) was
demonstrated by Euclid, Theorems Nine and Twenty-two; see Euclid, Optica,
ed. J. L. Heiberg (1895), pp. 166 and 180 (pp. 16 and 32); subsequently Aristot­
ete, Problemat 15.6, and Diogenes Laertius 9.89. This is applied more fre­
frequently to solid objects, for instance in the proposition that from a distance
four-cornered towers appear cylindrical: "φαίνουσα...τὸν πύρρον αἱ τετράγωνοι
στρόγγυλοι καὶ προσαπηγόντες πάροιδον ὑφόμενον" ("The square shapes of towers
appear cylindrical and falling forward when viewed from a distance"; "Aus­
züge aus Geminos," in Damian, Schrift über Optik, ed. Richard Schöne [Berlin, 1897],
p. 22, along with numerous parallel passages from Lucretius, Plutarch, Petron,
Sextus Empiricus, Tertullian and others). Later in the "Auszüge aus Geminos"
(p. 28) appears the following interesting perspectival explanation of entasis:
"ότι ποὺ τὸν μὲν καλλιόρισκαν κύκλα, ἐπὶ κατασκεύα (which should of course be
translated not as "broken" but as "weakened") ἔγειλε θεωρήσαν κατὰ μέσον πρὸς
ὅρων στενόμενον εὐρύτερον κατὰ τούτα ποιεί (sc. ὁ ἀρχιτέκτων) ("Thus, since a
cylindrical pillar as though weakened will appear more narrow at the middle,
the architect makes it wider at that point”); see Vitruvius 3.3.13. Vitruvius, too, calls for a curving of the horizontal architectural elements — again by way of compensation — in those eternally discussed passages 3.4.5 and 3.5.8 (see the survey of older views in the commentary, in itself quite unreliable, of Jakob Prestel, Zehn Bücher über Architektur des M. Vitruvius Pollio [Strassburg: Heitz, 1912-1914], vol. 1, p. 124). For the stylobate: “Stylobatum ita oparet essequari, uti habeat per medium adjectionem per scamillos impares; si enim ad libellum dirigetur, soleolatus aculo videhitur” (“It is best for the stylobate to be leveled, so that it has in its middle a projection through the use of leveling blocks of unequal heights. For it is constructed according to the book, it will seem hollowed-out to the eye”); and correspondingly for the epistyle and the capitals: “Capitis perfectis deinde columnarum non ad libellam, sed ad equelem modulum conlocatis, ut quae aedictio in stylobatis facta fuerit, in superioribus membris respondet, epistylorum ratio sic est habenda, uti…” (“Next, after the capitals of the columns have been constructed not according to the book, but built on an equal level, so that the same projection that will have been made in the stylobate has a corresponding projection in the members above, the proportion in the epistyles is to be similarly made so that…”). The apparently correct reading of the first passage, that of Émile Burnouf (“Explication des courbes dans les édifices doriques grecs,” Revue générale de l’architecture 32 [1875], cols. 145-53; adopted by William H. Goodyear, Greek Refinements [New Haven: Yale University Press, 1912], p. 114) seems to have been unfairly disregarded by German scholars: the scamilli (literally, “little stools”) are not supports for the columns — this would produce not a swelling of the stylobate, but rather only a swelling of the succession of bases — but rather leveling blocks (nivellettes) which had been placed on the cut stones to facilitate gauging. If these leveling blocks are “unequal,” that is, if they diminish in size toward the middle, this will in fact produce the convexly curved stylobate described by Vitruvius (Figure 11).

All these remarks prove that the ancients were familiar with visual curvatures, and that they were able to explain certain architectural motifs to themselves only as efforts to optically neutralize these curvatures. And if this explanation, considered purely from the viewpoint of art history, seems incom-
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FIGURE 11. Explanation of Vitruvius's *scamilli impares* (after Burnouf): below, the stylobate is leveled by means of equal leveling blocks, such that the stones form a straight line; above, the stylobate is leveled by means of blocks whose size decreases toward the center (*scamilli impares*), creating an upward curve (*per medium adiecto*).

elusive or at any rate one-sided, then it is all the more striking how important those curvatures were to antique artistic theory. There is, however, a curious problem: the architectural curvatures that Vitruvius speaks of behave exactly oppositely to what one would expect in light of their documented purpose, namely to counteract the visual curvature. Moreover, when these curvatures can actually be verified (the most important case is the Parthenon), Vitruvius's claims are usually borne out. Whereas one might suppose that the convexity of the visual curves would be canceled out by a concavity of the architectural curves, the raising of the middle of the stylobate and epistyle produces exactly, on the contrary, a bulging upward of the horizontal (the same effect can also be obtained through a convex arching of the facade in the ground plan, as at Nîmes and Paestum). Guido Hauck's explanation of this phenomenon by the so-called corner-triglyph conflict, or rather by the diminishing of the spaces between the lateral columns which were supposed to alleviate the corner-triglyph conflict (*Die subjektive Perspektive*, p. 93ff.), has been invalidated by the discovery of curvatures even on non-Doric temples, where naturally such a corner-triglyph conflict cannot take place. G. Giovannoni attempted to replace this refuted explanation in "La Curvatura delle linee nel Tempio d'Ercole a Cori," *Mitteilungen des Deutschen Archäologischen Instituts, Römische Abteilung*.
23 (1908), pp. 109-30. Our consciousness, he argued, is so alert to the contrast between perspectival appearance and objective reality that, in a sense, it overcompensates for the perspectival modifications; that is, accustomed to regarding the objectively false as correct, we in many cases perceive the objectively correct as false. Exactly cylindrical columns which, in a physiological sense, ought to appear to diminish toward the top, would be psychologically perceived to expand toward the top, for perspectival convergence ordinarily undergoes such a strong overcompensation that only a still-stronger convergence, that is, an objectively somewhat conical structure, will produce the impression of a pure cylindrical form. And likewise the apparent convexity of straight lines would be so strongly overcompensated that we would perceive the real straight lines as concave, and accordingly, in an apparent paradox, would receive an impression of real straightness only when the lines were in fact convex.

As complicated and, as it were, acrobatic this explanation appears, it nevertheless looks like the most plausible, albeit only under the assumption of an almost unimaginably sensitive and elastic sense of form. It is perhaps even corroborated, even if only indirectly, by the passage just quoted from Geminos about towers. If it says there that towers seen from afar seem to "fall forward," then it follows that antique consciousness must in fact have been accustomed to carrying out a kind of overcompensation in Giovannoni's sense. For in and of itself perspectival foreshortening of vertical structures, when it is very strong (for example at close range), produces the impression of falling backward; thus Vitruvius's precept about the inclining of cornices (3.5.13). And when the foreshortening is weak (for example at long range), and when therefore a normal impression of objective verticality might justifiably be expected, then the illusion of falling forward can only be the effect of overcompensation.

It should, however, not be concealed that those very curvatures of the temple of Hercules at Cori upon which Giovannoni built his theory have recently turned out to be merely accidental: see the apparently conclusive presentation by Armin von Gerkan, "Die Krümmungen im Gebälk des dorischen Tempels in Cori," Mitteilungen des Deutschen Archäologischen Instituts, Römische Abteilung 40 (1925), pp. 167-80.

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It is most interesting that antique theory, when it asserts that an angle appears rounded when seen from afar, finds itself in accord with the most recent psychological research: H. Werner has proven that the less one conceives of an angular structure as "articulated," that is, the more one perceives the "angle" as the mere interruption of a single form and not as the encounter of two forms, then the more the structure undergoes a kind of polishing off or rounding ("Studien über Strukturgesetze," Zeitschrift für Psychologie 94 [1924], p. 248ff.). This phenomenon appears, for example, when a broken line is drawn over and over again by an experimental subject instructed to maintain an "integral" perception; but also when unclear vision – for example at a great distance – hinders the "articulated" and favors the "integral" perception. If, on the contrary, the subject is held to the "articulated" perception, then there emerges an increasing tendency toward a concave sharpening of the corner. This would be the case at Segesta, where the tapering off of the facade preserves the clear "articulation" of the building from the rounding off that would otherwise threaten when seen from a distance.

This also sheds light on a curious phenomenon in medieval miniatures (the best-known examples are the Reichenauer manuscripts in Munich, Bayerische Staatsbibliothek, Clm. 57, 58, 59, published by Georg Leidinger in the series Miniaturen aus Handschriften der Kgl. Hof- und Staatsbibliothek in München), namely, when the prismatic crib in which the Christ child lies (or towers or other such objects) is represented in the form of Figure 12A. This peculiar rounding off of the rear corner is evidently explained by the failure of the medieval artists to understand the perspectivally foreshortened forms of their (presumably early Christian) models. In this case the psychological obscuring of the conception of form favors the suppression of the "articulated" interpretation, just as did the physical obscuring of the perception of form when it was seen from a great distance. The acute angles of the original form (Figure 12B) were naturally preserved from the rounding off; but also the obtuse angle at the front (b) was, in a sense, spared by the vertical joining it. Thus the rounding off would be limited to the rear angle (a), unless the prismatic object is covered by a cloth concealing its lower corner: for in that case this corner too can be
affected by the rounding off (see, e.g., Cim. 57 [= cod. lat. 4452], pl. 28).

13. See Damian, Schrift über Optik, p. 2, art. 11: "ὅτι ἡ τοῦ ἄνω κόνου κορυφή ἐνος ἐκ τῆς κόρως καὶ κύκλων ἐκείνα φαινέσθων..." ("The point of the cone of vision is within the pupil and it is the point of a sphere...") (see also p. 8ff.). From this it becomes immediately evident that it is one and the same form of thought — or better, form of seeing — which, on the one hand, makes the visual magnitudes so strictly dependent on the angles, and on the other hand, emphasizes so strongly the apparent curvature of the straight lines.

14. Euclid, Definition (hara) 4–6, Optica, p. 154 (p. 2).

15. Euclid, Theorem Eight, Optica, p. 164 (p. 14): "τὰ Ἡπα μεγέθη ἄνω καὶ ὑποτικών σεικ ἀνάλογα τῶν ἀποστάσεων άραντι..." ("Two objects of equal magnitude placed at unequal distances are not seen according to the ratio of their distances"). The proposition is proven by showing that the difference between the distances is greater than that between the angles, and that only the latter (according to the axioms named in the previous note) determine the visual magnitudes.

16. Jean Pélerin (Viator), De artificiali perspectiva (Toul, 1505), facsimile ed., by A. de Montaiglon (Paris, 1860), fol. C8r. The corresponding illustration, which is of course dependent on Dürer’s Martydom of the Ten Thousand, first appears in the edition of 1509. (For the explanation, see Figure 4.)

It is very instructive that Leonardo, with respect to the diminishment of distances, arrived “per isperienza” at the very result to which linear perspectival construction leads: namely, to the notion that the apparent magnitudes of equal sections are inversely proportional to their distances from the eye (Das Buch von der Malerei, Quellenschriften für Kunstgeschichte und Kunsttechnik des
Here, evidently, linear perspectival thinking showed the way for concrete observation, and in fact Leonardo speaks even here, where he formulates an "empirically" discovered law of perspectiva naturalis, of a "picture surface" ("pariete"). No matter whether he mentally projected objects onto this picture surface, or whether (and this seems more probable) he in fact made observations with the help of that apparatus with the pane of glass which he knew so well, and which he recommends also for the corresponding observations of color (attenuation of local color in objects at distances of 100, 200, 300 braccia, etc.); see Das Buch von der Malerei, art. 261 and Richter, no. 294. Thus the establishment of this law in no way constitutes "progress" beyond geometrical perspectival construction (as Heinrich Brockhaus claims in his worthy edition of Pomponius Maitritius, De sculpture [Leipzig, 1880], p. 47ff.), but rather only an unconscious application of its results to the direct observation of objects: in a sense, a repercussion of perspectiva artificialis upon perspectiva naturalis.

In other situations, however, for example with natural objects (that is, where there is no question of a planar projection), antique angular perspective reasserts itself, even in the Renaissance. Dürer, for example, in order that lines of script written on a wall all appear to be of the same height, recommends enlarging them as they rise, such that the respective visual angles are all equal (Figure 13: \(a\) appears to equal \(b\) which appears to equal \(c\), if \(a = \theta = y\); Unterweisung der Messung [1525], fol. k10). This is in keeping with Euclid's Theorem Seven and with the often-attested practice of antique sculptors of permitting the proportions of a figure in a high place to increase toward the top in order to counteract

![Figure 13](image-url)
the contraction brought about by the diminishment of the visual angle. See
Daniello Barbaro, La Pratica della prospettiva (Venice, 1569), p. 9 (with explicit
reference to Dürer); Athanasius Kircher, Ars magna lucis et umbrae (Rome, 1646),
p. 187ff. (with the example of Trajan's column and a citation from Vitruvius,
6.2.lff., where the same "detractiones et adiectiones" are discussed); or Sandrart in
his Teutsche Academie (Nuremberg, 1675), 1.3.15, p. 98. Even Leonardo, in his
doctrine (mentioned in note 8, above) of the "natural" foreshortenings that the
margins of any picture undergo, tacitly presupposes the angle axiom. Indeed, per­spective naturalis in general is almost entirely governed by the angle axiom, even
when it serves as an introduction to treatises on artificial perspective (as is the
case with Barbaro, Serlio, Vignola-Danti, Pietro Cataneo, Aguilonius and others).
It was, however, customary either to ignore Euclid's Eighth Theorem, or to dis­
arm it by emending the text; for on account of its reference to the diminishment
of magnitudes into depth, it stood too unequivocally in contradiction to the
rules of perspectiva artificialis (see the following note). Indeed, it can even be
stated that the Renaissance, at least as far as perspectiva naturalis is concerned,
was almost more rigorously Euclidean than the Middle Ages, which knew Euclid
only from the Arabic tradition, already somewhat modified. Roger Bacon, for
example (Perspectiva 2.2.5; p. 116ff. in the Frankfurt edition of 1614), precisely
following Alhazen (Optica 2.36ff.; p. 50ff. in Risner's Basel edition of 1552),
teaches that the visual angle alone does not determine the perception of magni­tudes; rather, magnitudes are estimated only by a comparison of the object (that
is, the base of the visual pyramid) with the visual angle and the distance of the
object from the eye; this distance is itself assessed on the basis of the empirically
familiar magnitudes of the intervening objects. And in Vitellio (Perspectiva com­
munis 4.20; p. 126 in Risner's edition of 1552) we find even this: "Onne quod sub
maiori angulo videtur, maius videtur, et quod sub minori minus: ex quo potest, idem
sub maior angulum vidum apparec maius se ipso sub minori angulo viso. Et univer­saliiter secundum proportionem anguli fit proportio quantitatis rei directe vel sub eodem
obliquitate visae; in oblique tamen visum vel in his, quorum unusum videtur directe,
alterum oblique, non sit" ("Anything that is seen at a greater angle appears greater,
and anything seen at a lesser angle appears less great. In what appears, the same
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thing seen at a greater angle appears greater than the same thing viewed at a lesser angle. And in general the proportion of size of a thing comes about directly according to the proportion of the angle or at the same angle of vision...; nevertheless in the case of things seen from the side or in the case of those in which one thing is seen from the side and another straight on, this is not so”). In Figure 14A, therefore, the apparent sizes are proportional to the visual angles; in Figures 14B and 14C, by contrast, they are not. The justification that Bacon offers for this interpretation, which departs from Euclid, is, interestingly, a purely psychological one: a square is seen from an angle such that $COB > BOA$ (Figure 15), and yet the sides $AB$ and $BC$ are perceived as equal—an impression that indeed can only be explained by the mind’s “stabilising tendency” (see p. 31). It can be seen, however, that the critique of Euclid carried out by medieval perspectiva naturalis was entirely differently motivated than that of the modern perspectiva artificialis, according to which—in the case just discussed—the difference of the apparent magnitudes (namely, the projections $AB$ and $BC$ onto a straight line $XY$) must be even greater than the difference between the angles $COB$ and $BOA$.

17. The metamorphosis of Euclid’s Eighth Theorem (at least when it is not simply dropped, as it is in most writings on artificial perspective) can be followed almost step by step. The first complete published translation, that of Zamberto (Venice, 1503), still renders it literally, even if a little misleadingly by virtue of placing “intervallis” before “proportionaliter”: “Aequales magnitudines inaequaliter expositae intervallis proportionaliter minime spectantur” (“Equal dimensions set unequally at distances appear least proportionately”; fol. A A.v.). Dürer, or the Latinist he relied upon, immediately fell victim to the ambiguity of this translation, in that he referred “proportionaliter” to “expositae” instead of to “spectantur,” and “minime” to “spectantur” instead of to “proportionaliter,” thus rendering the sentence perfectly unintelligible: “Gleich Gross ungleich gesetzt mit proportionalen Unterscheiden können nit gesehen werden” (“Equal dimensions set unequally with proportional differences cannot be seen”), Lange and Fuhse, Dürers schriftlicher Nachlass, p. 322, l. 23 (indeed, the whole passage from Lange and Fuhse, p. 319, l. 21 to p. 326, l. 19 is a translation from Euclid: see Panofsky, Dürers Kunsttheorie [Berlin: Reimer, 1915], p. 15ff.). The standard translation for
the entire subsequent period, that of Johannes Pena (Paris, 1557, p. 10; 1604, p. 8) emends it thus: "Aequales magnituidines inaequaliter ab oculo distant, non servent eandem rationem angulorum quam distantiam" ("Equal dimensions standing at unequal distances from the eye do not have the same ratio of angles as their distances"). Both the Italian translation of Ignazio Danti, _La Prospettiva di Euclide_ (Florence, 1573), p. 27, and the French translation of Roland Fréart de Chantalou, _La Perspective d'Euclide_ (Le Mans, 1663), p. 19, follow Pena exactly. Thus the premise that the angles are not proportional to the distances is made into the conclusion, whereas the actual conclusion, namely the proposition that the ratio of the apparent sizes is determined only by the ratio of the angles and not the distances, is simply omitted. Since Euclid's proof is adopted unaltered, this is in fact a _demonstratio per demonstrandum_.

18. Vitruvius, in the passage in question (1.2.2; on its much-disputed significance for antique perspectival construction, see the following note), takes the term _scenographia_ in its narrower sense as the method of representing buildings perspectivally on a surface, whether for architectonic or theoretical purposes: _ichnographia_ means the representation of the building in plan, _orthographia_ means the elevation, and _scenographia_ means a perspectival display that shows the sides as well as the facade ("frontis et laterum abscendentium adumbratio"); see also the parallel passage, 7, Prooemium, cited in the following note). But the term _scenographia_ also has a broader sense, for it can denote quite generally the application of optical laws to the visual arts and architecture in their entirety; that is, not only the rules for making flat pictures on flat surfaces, but also the rules of architectonic and plastic construction, insofar as the latter are interested in counteracting the distortions entailed in the process of seeing (see notes 12 and 16, above). This definition is most clear and complete in Geminos ("Auszüge aus Geminos," in Damian, _Schrift über Optik_, p. 28):

_Tί ἐστι σκηνογραφικὸν._

Τὸ σκηνογραφικὸν τὴν ὁπισθεῖς μέρος ὑπὲρ τὸς προσάκης γράφειν τὰς εἰκόνας τῶν αἰκονομημάτων. ἐπειδὴ γὰρ ὅπως ὁλα[τε] ἦν ὁ ὅντα, τοιαῦτα καὶ φαίνεται,